CRITICAL LOAD OF A MICROPILE CONSIDERING SOIL LIQUEFACTION

Zhu Yuanyuan[1] and Fusanori Miura[2]

[1]Graduate School of Science and Engineering, Yamaguchi University, Ube, Japan
[2]Faculty of Engineering, Yamaguchi University, Ube, Japan

In this paper, we first set up a mathematical model of a single pile subjected to axial load with initial horizontal displacement in elastic soil, then obtained critical loads and expressed them in nondimensional form. Based on the results from parametric study, we found that the critical load decreased with the increment of the initial displacement and the effect of it is very small, but the effect of the reduction of soil spring due to liquefaction on the decrease of the critical load is significant.

Key Words: pile foundation, stability analysis, critical load, nonlinear system

1. INTRODUCTION

According to recent post-earthquake investigation reports, we can see plenty of damage patterns of piles and pile foundations[1][2]. Some of them were caused by strong ground motions and some of them by liquefaction-induced large ground displacements. When a pile foundation is subjected to liquefaction-induced large ground displacement, the behavior of the foundation is very complex because of geometric nonlinearities and material nonlinearities of both pile and soil. In other words, so-called $P-\Delta$ effect due to lateral ground displacement, reduction of the rigidity of flexure of the pile due to large deformation, and reduction of soil spring coefficient due to the occurrence of liquefaction. From the purpose of design, we need to understand the ultimate state of the pile foundation under such conditions, and to do so, we need to analyze the stability of the pile foundation.

To attempt to analyze the stability of the pile foundation, we must calculate the critical load of a pile at first based on the stability theory. During past several decades, some researchers, such as Bjerrum[3], Mascardi[4], Mandel[5], Fleming[6] , et al., studied the critical load of pile for different types. Their results present useful information for the purpose of design of pile foundation, however, they did not consider the pile deformation. In this paper, we study the critical load of a pile with initial deformation in elastic ground as a first step of the nonlinear stability analysis of pile foundation.

Based on the assumption that the surrounding ground is expressed as distributed soil springs, we first set up the geometrically nonlinear mathematical model of a single pile with initial deformation. Then by solving the problem, we obtained critical loads in nondimensional form. By applying the actual values of diameter, length, flexure of rigidity of a pile and N-values of ground etc. to the nondimensional solution, we performed parametric study to investigate the effect of the magnitude of initial deformation and the reduction of the soil spring due to liquefaction on the critical load.

2. GOVERNING EQUATION

We assume that the pile foundation has initial deformation of $u_0(s)$ in x direction and $w_0(s)$ in y direction as shown in Fig.1. The domain occupied by the pile at the initial state before additional deformation due to the application of axial load P occurs is expressed as

$$\Gamma_0 : \{(x, y) | x = s + u_0(s),\ y = w_0(s),\ 0 \le s \le l\}$$

in which, $s \in [0, l]$ is the coordinate along the pile axis and l is the length of the pile (see Fig. 1).

$\theta_0(s)$ is an angle between tangent line and x-axis at Point C. From Fig.2, we can get the following geometric relations,

$$\begin{aligned} \cos\theta_0(s) &= \frac{ds + du_0(s)}{ds} = 1 + u_0'(s) \\ \sin\theta_0(s) &= \frac{dw_0(s)}{ds} = w_0'(s) \end{aligned} \tag{1}$$

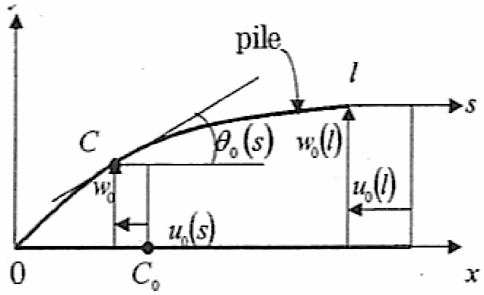


Fig.1 Initial deformation of a pile and the coordinate systems

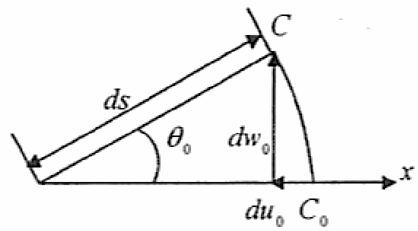


Fig.2 Geometric relation of the infinitesimal pile length at the initial state

Equations (1) are nonlinear because there are nonlinear terms , $\cos\theta_0(s)$, $\sin\theta_0(s)$, in the equations. It should be noted that $1 = 1 + u_0'(s) \Rightarrow u_0'(s) = 0$ and $\theta_0(s) = w_0'(s)$ in the linear problem.

Hereafter, we express u_0, w_0 etc. instead of $u_0(s)$, $w_0(s)$ etc. for the simplicity unless misunderstanding occurs.

Let $q(s)$ be the reaction of soil, and the pile is subjected to axial external force P at the point of $s = l$. After applying the force, an arbitrary point $C(s+u_0,\ w_0)$ in the Γ_0 moves to point $C'(s+u_0+u,\ w_0+w)$, and the pile occupies the new domain of

$$\Gamma:\{(x,y)|x = s+u_0+u,\ y = w_0+w\ ,0 \le s \le l\}$$

in which, u and w are additional displacements in x-direction and y-direction caused by P respectively.

(1) Geometric relationship

We have following nonlinear geometric relationships in the same way as equation (1).

$$\begin{aligned}\cos\theta &= 1+u_0'+u' = \cos\theta_0+u' \\ \sin\theta &= w_0'+w' = \sin\theta_0+w'\end{aligned} \tag{2}$$

in which, $\theta(s)$ is an angle between tangent line and x-axis (see Fig.3).

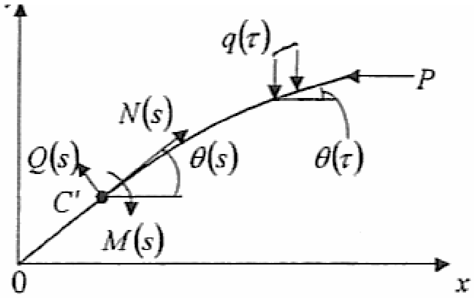


Fig.3 Notations of pile after deformation caused by axial external force P

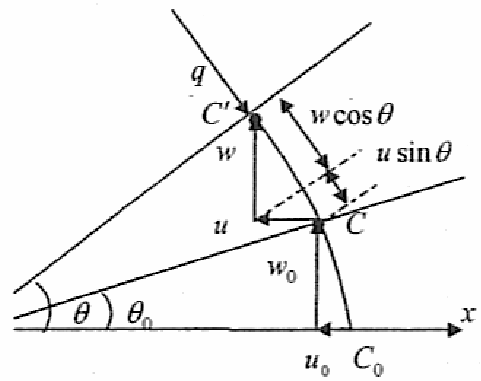


Fig.4 The reaction of soil q

(2) Equilibrium equation

The soil reaction $q(s)$ is obtained by multiplying the soil spring stiffness k and deformation of the pile at point $C'(s+u_0+u,\ w_0+w)$. By assuming that the direction of $q(s)$ is normal to the pile after application of the force (see Fig.4), we get

$$q(s) = k\{u(s)\sin\theta(s) + w(s)\cos\theta(s)\} \tag{3}$$

in which, the spring stiffness k is determined by multiplying the reaction coefficient k_h and pile diameter D as commonly done.

From the equilibrium at point C' in the transverse direction of the pile (see Fig.3), we get

$$Q(s) + P\sin\theta(s) - \int_s^l q(\tau)\cos(\theta(s)-\theta(\tau))d\tau = 0 \tag{4}$$

that is

$$\begin{aligned}&Q(s)+P\sin\theta(s) - k\int_s^l (w(\tau)\cos\theta(\tau) \\ &+u(\tau)\sin\theta(\tau))\cos(\theta(s)-\theta(\tau))d\tau = 0\end{aligned} \tag{5}$$

in which, $Q(s)$ is the shear force at point C'.

(3) Constitutive equation

The constitutive equation is given as

$$Q = EI(\theta'' - \theta_0'') \tag{6}$$

in which, EI is the flexural rigidity of the pile.

(4) Boundary condition

We assume that both sides of the pile is simply supported (S.S), in which the boundary conditions are

$$\begin{cases} u(0) = w(0) = 0, \ \theta'(0) = \theta_0'(0) = 0 \\ w(l) = 0, \ \theta'(l) = \theta_0'(l) = 0 \end{cases} \tag{7}$$

The nonlinear boundary value problems are eqs. (2), (5), (6) and (7).

Substituting constitutive equation (6) into equilibrium equation (5) of the pile, and by differentiating eq.(5) with respect to s, we can get

$$\begin{aligned} &EI(\theta''' - \theta_0''') + P\cos\theta\theta' + k[u\sin\theta + w\cos\theta] \\ &- k\int_s^l (w(\tau)\cos\theta(\tau) + u(\tau)\sin\theta(\tau))\sin(\theta(\tau) - \theta(s))d\tau \\ &\times \theta'(s) = 0 \end{aligned} \tag{8}$$

From the differentiation of the second equation of eq.(2) by s, we obtain

$$\cos\theta\theta' = w'' + w_0'' \tag{9}$$

When the problem is limited in small deformation, that is, $w(s)$, $u(s)$, $\theta(s) - \theta_0(s)$ are small, from eq. (2), we get $u' = \cos\theta - \cos\theta_0 \to 0$ and $w' = \sin\theta - \sin\theta_0 \to \theta - \theta_0$. Furthermore, we get $u = 0$ because of $u(0) = 0$ and $u' = 0$.

We can assume that $w(s) = o(\varepsilon)$, $u(s) = o(\varepsilon)$, and $\theta(s) = o(\varepsilon)$, so we get

$$\begin{aligned} &k\int_s^l (w(\tau)\cos\theta(\tau) + u(\tau)\sin\theta(\tau))\sin(\theta(\tau) - \theta(s))d\tau \\ &= o(\varepsilon^3) \end{aligned}$$

Thus, we can neglect the integral term in the equation (8) for simplicity of the problem.

Substituting eq.(9), $w' = \theta - \theta_0$ and $u = 0$ into eq.(7) and eq, (8), thus, we have boundary value problem as follows;

$$\begin{aligned} &EIw^{(4)} + Pw'' + k\cos\theta_0 w = -Pw_0'' \\ &w(0) = w''(0) = w(l) = w''(l) = 0 \end{aligned} \tag{10}$$

The following non-dimensional parameters are introduced.

$$\begin{cases} t = \dfrac{s}{l}, \quad W = \dfrac{w}{l}, \quad U = \dfrac{u}{l}, \\ \lambda = \dfrac{Pl^2}{EI}, \quad \alpha = \dfrac{kl^4}{EI} \end{cases} \tag{11}$$

Substituting eq.(11) into eq.(10), the non-dimensional boundary value problem becomes;

$$\begin{aligned} &W^{(4)} + \lambda W'' + \alpha\cos\theta_0 W = -\lambda W_0'' \\ &W(0) = W''(0) = W(1) = W''(1) = 0 \end{aligned} \tag{12}$$

The problems (12) are valid under the condition that the initial deflection may be large, but the additional deformation due to axial external force is small, i.e., $w(s)$, $u(s)$, $\theta(s) - \theta_0(s)$ are small.

3. CRITICAL LOAD OF PILE

In this chapter, we will discuss the critical load obtained from problem (12), then give the limit of critical load. The critical load that we get is that of pile with initial deformation, which is different from other researchers.

Let $\alpha_\theta = \alpha\cos\theta_0$, which is the given constant, the characteristic values and corresponding characteristic functions of eq.(12) are

$$\begin{aligned} &\lambda_m = m^2\pi^2 + \frac{\alpha_\theta}{m^2\pi^2}, \quad (m = 1, 2, \ldots) \\ &W_m = \sin m\pi \end{aligned} \tag{13}$$

Let

$$\lambda_{cr} = \min\{\lambda_m\} = \lambda_{m^*} \tag{14}$$

then from the eqs. (11) and (13), the critical load and the corresponding characteristic function are

$$\begin{aligned} &P_{cr} = \frac{EI}{l^2}\lambda_{cr} \\ &w_{cr}(s) = \sin\frac{m^*\pi}{l}s \end{aligned} \tag{15}$$

We can see from the above equations that the critical load is the function of m^*, and m^* is the function of α_θ. We will discuss the value of m^* according to the magnitude of α_θ as follows.

According to eq. (13), if we set function

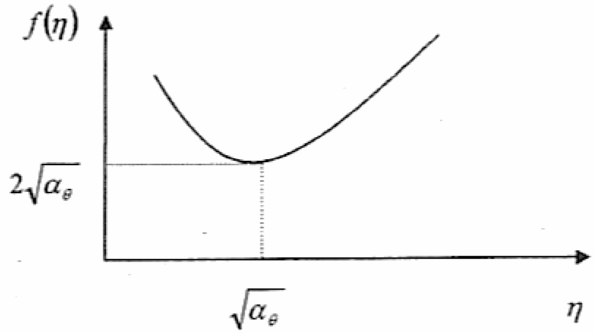


Fig. 5 Minimum value of $f(\eta)$

$f(m^2\pi^2)=m^2\pi^2+\frac{\alpha_\theta}{m^2\pi^2}$, the minimum value of f is $2\sqrt{\alpha_\theta}$ at the point of $m^2\pi^2=\sqrt{\alpha_\theta}$ (see Fig.5). In our problem, $f(m^2\pi^2)=\lambda_m$. So our object is finding suitable integer m^* that makes $f(m^2\pi^2)=\lambda_m$ minimum.

When $(m^*\pi)^2=\sqrt{\alpha_\theta}\le\pi^2$, we have $m^*=1$ certainly.

When $(m^*\pi)^2=\sqrt{\alpha_\theta}\le(2\pi)^2$, that is, m^* is 1 or 2, we still have $m^*=1$ if $f(\pi^2)\le f(4\pi^2)$ because smaller function value, $f(\pi^2)$, is the critical load which corresponds to $m^*=1$. From $f(\pi^2)\le f(4\pi^2)$, i.e., $\pi^2+\frac{\alpha_\theta}{\pi^2}\le 4\pi^2+\frac{\alpha_\theta}{4\pi^2}$, we get $\sqrt{\alpha_\theta}\le 2\pi^2$ by subtracting the left term from the right term. So we can get conclusion that $m^*=1$ when $\sqrt{\alpha_\theta}\le 2\pi^2$.

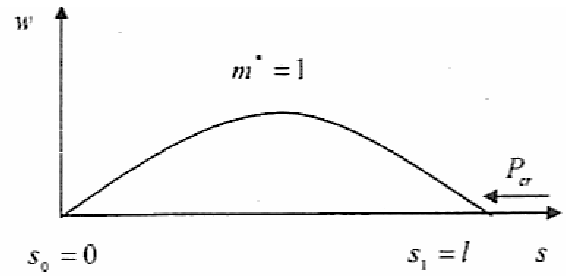
Accordingly, when $2\pi^2<(m^*\pi)^2=\sqrt{\alpha_\theta}\le 4\pi^2$, we have $m^*=2$.

When $(2\pi)^2\le(m^*\pi)^2=\sqrt{\alpha_\theta}\le(3\pi)^2$, that is, m^* is 2 or 3, we still have $m^*=2$ if $f(4\pi^2)\le f(9\pi^2)$. From $f(4\pi^2)\le f(9\pi^2)$, we get $\sqrt{\alpha_\theta}\le 6\pi^2$ in the same way as above. So we can get conclusion that $m^*=2$ when $2\pi^2\le\sqrt{\alpha_\theta}\le 6\pi^2$.

We can get following conclusion by analogy.

$$m^*(\alpha_\theta)=m,\quad \text{when}\quad m(m-1)\pi^2<\sqrt{\alpha_\theta}\le m(m+1)\pi^2 \qquad m\ge 2 \qquad (16)$$

By using $\alpha_\theta=\frac{kl^4}{EI}\cos\theta_0$ in eq. (16), we obtain the following relations:



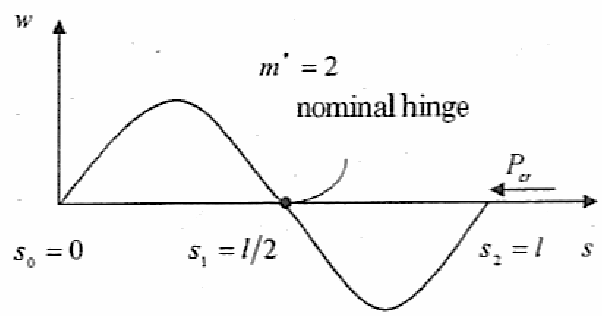


Fig. 6 The nominal hinge of pile

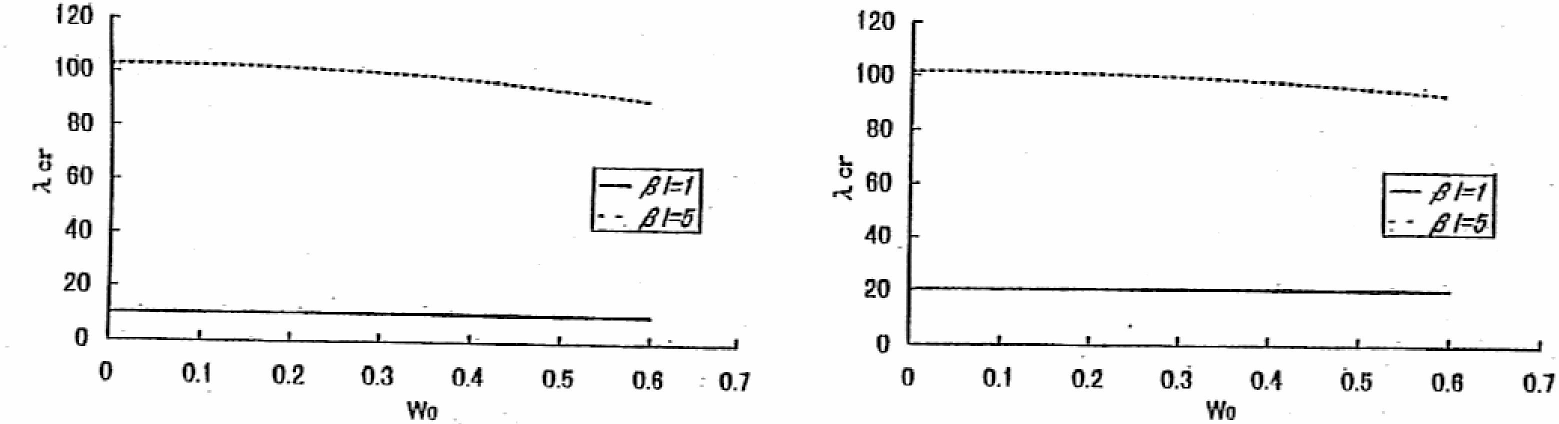
$$m^*(l)=m,\quad \text{when}\quad m(m-1)\pi^2\sqrt{\frac{EI}{k\cos\theta_0}}<l^2\le m(m+1)\pi^2\sqrt{\frac{EI}{k\cos\theta_0}} \qquad m\ge 2 \qquad (17)$$

When $k=0$, $\alpha_\theta=0$, then, $m^*=1$, $\lambda_{cr}=\pi^2$, and $P_{cr}=EI\pi^2/l^2$, which is the Euler Critical load. The corresponding characteristic function is $w_{cr}(s)=\sin\frac{\pi}{l}s$, it has only one maximum value as shown in Fig.6.

When $k>0$, $\alpha_\theta>0$, m^* changes corresponding to l from eq.(17), that is $m^*=m^*(l)$. If l is enough long, we can assume that $m^*\ge 2$ because larger m^* is required for longer l to satisfy eq. (17). Let $[0,l]$ be equally divided into m^*, the points which cross s-axis as shown in Fig. 6 are $s_i=\frac{i}{m^*}l$, $(i=0,1,\dots m^*)$. They give $w_{cr}(s_i)=0$ and $w''_{cr}(s_i)=0$, and correspond to the nominal hinge.

It should be pointed out that $P_{cr}(l)$ is the critical load of the following problem in which the length of the pile is $\frac{l}{m^*}$.

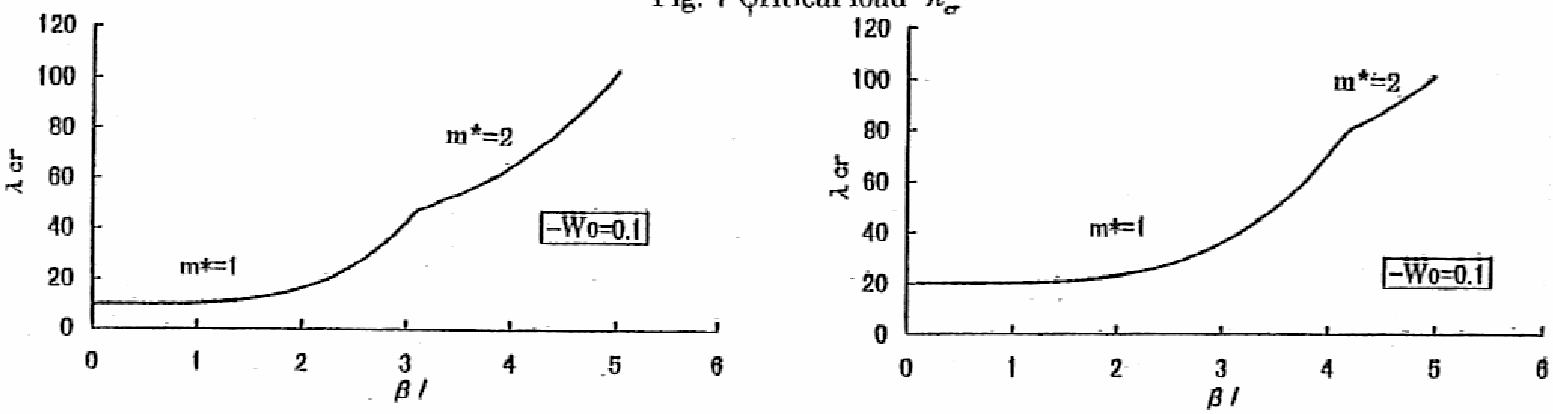
$$\begin{aligned} &EIw^{(4)}+Pw''+k\cos\theta_0 w=-Pw_0'' \\ &w(0)=w''(0)=w(s_1)=w''(s_1)=0 \end{aligned} \qquad (18)$$



(a) S.S. type

(b) C.S. type

Fig. 7 Critical load λ_{cr}



(a) S.S. type

(b) C.S. type

Fig. 8 The relationship between λ_{cr} and βl

We can calculate the critical load $P_{cr}(l)$ by using eq.(18) for the long pile surrounded by soil.

From eq.(17), it gets

$$\sqrt{1-\frac{1}{m^*}}\cdot\sqrt[4]{\frac{EI}{k\cos\theta_0}}\pi \le \frac{l}{m^*} \le \sqrt{1+\frac{1}{m^*}}\cdot\sqrt[4]{\frac{EI}{k\cos\theta_0}}\pi \tag{19}$$

It shows that the equivalent length of the pile, $\frac{l}{m^*}$, of the problem (18) is less than $\sqrt{1+\frac{1}{m^*}}\cdot\sqrt[4]{\frac{EI}{k\cos\theta_0}}\pi$, and because $m^*\to\infty$ when $l\to\infty$, we can get

$$\lim_{l\to\infty}\frac{l}{m^*(l)}=\sqrt[4]{\frac{EI}{k\cos\theta_0}}\pi \tag{20}$$

Furthermore, we get

$$\begin{aligned}\lim_{l\to\infty}P_{cr}(l)&=\lim_{l\to\infty}\frac{EI}{l^2}\lambda_{cr}\\&=\lim_{l\to\infty}\frac{EI}{l^2}\left(m^{*2}\pi^2+\frac{\alpha_\theta}{m^{*2}\pi^2}\right)\\&=\lim_{l\to\infty}\left(EI\pi^2\frac{m^{*2}}{l^2}+\frac{k\cos\theta_0 l^2}{m^{*2}\pi^4}\right)\end{aligned} \tag{21}$$

By substituting eq. (20) into eq. (21), we get

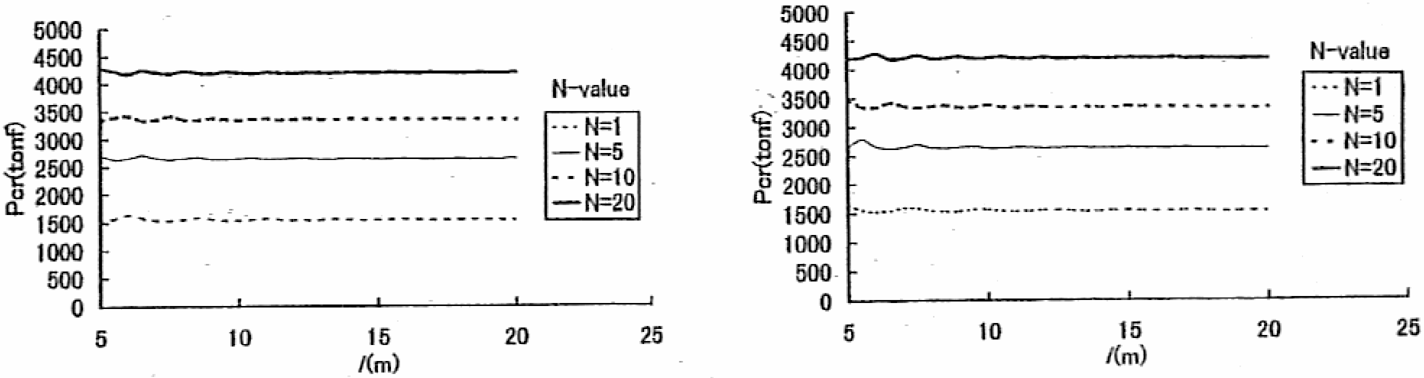
$$\lim_{l\to\infty}P_{cr}(l)=2\sqrt{EIk\cos\theta_0} \tag{22}$$

Eq. (22) shows that $\lim_{l\to\infty}P_{cr}(l)=0$ when there is no soil reaction in the problem (10), that is, $k=0$.

4. EXAMPLE

We choose one type of pile of which diameter is 177.8mm. The soil spring stiffness k is determined based on the specifications for highway bridges [7], that is

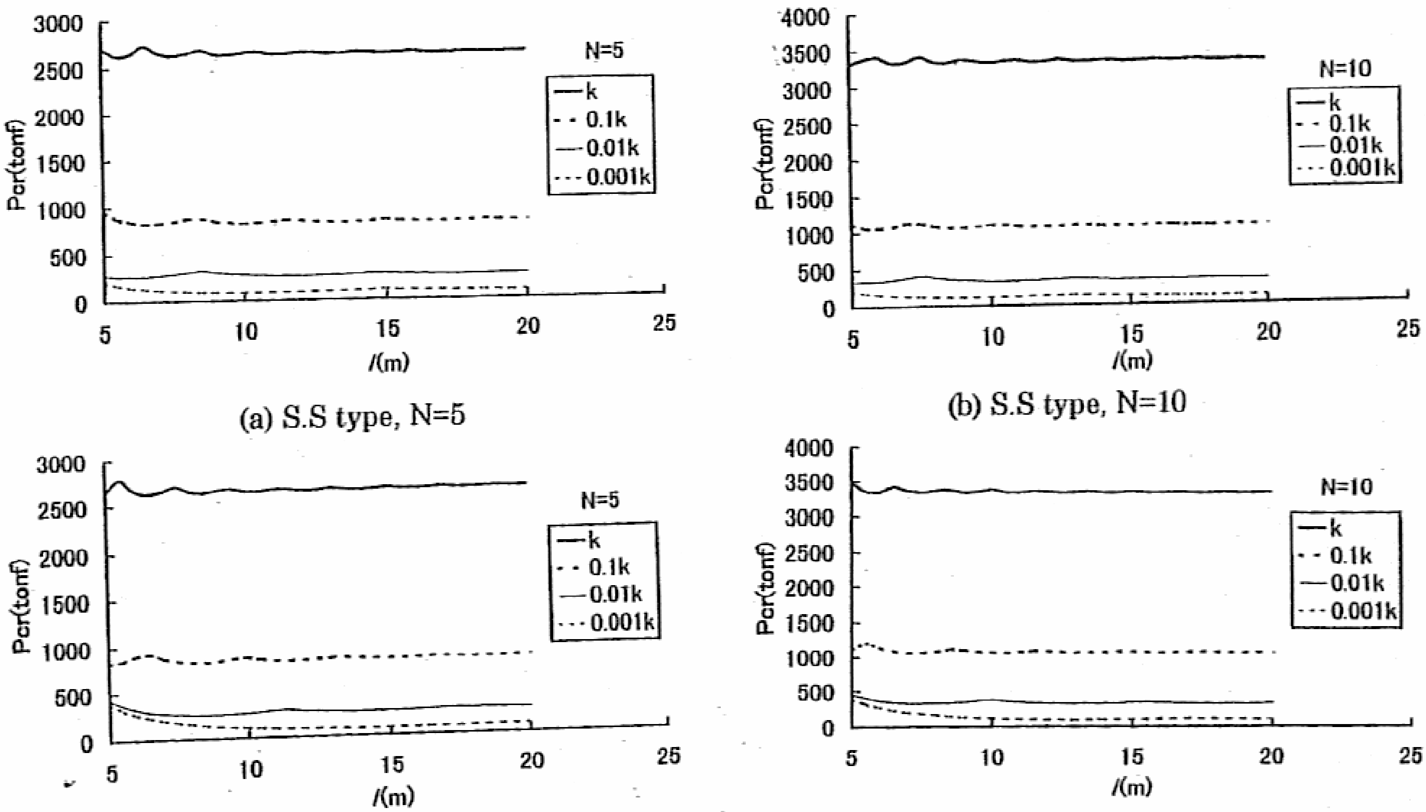
$$k=k_H D \tag{23}$$



(a) S.S type, w_0=1m

(b) C.S type, w_0=1m

Fig. 9 The relationship between l and P_{cr}



(a) S.S type, N=5

(b) S.S type, N=10

(c) C.S type, N=5

(d) C.S type, N=10

Fig. 10 The liquefaction of pile

in which, k_H is the reaction coefficient and D is the diameter of the pile.

k_H can be calculated from the following equations:

$$k_H = \frac{1}{30} E_D$$

$$E_D = 2(1+\upsilon_D)G_D$$

$$\upsilon_D = 0.5, \quad G_D = \frac{\gamma_t}{10g} V_{SD}^2$$

$$\gamma_t = 1.7, \quad V_{SDi} = c_V V_{si}$$

$$c_V = 0.8, \quad V_{si} = 80 N_i^{1/3}$$

The parameters of the pile used in the parametric stud are shown in Table 1.

We get the dimensionless critical load λ_{cr} of the piles a shown in Fig.7 from eq. (12), in which, $w_0(0)=0$ $\theta_0(s)=const$. As another example, we also calculated th critical load of a pile of which one side is simply supported,

Table 1. Parameters of a micropile

External Diameter D(mm)	Internal Diameter d(mm)	Flexural Rigidity EI(tf.m²)	Length l (m)
177.8	152.5	473	15

but the other is clamped (C.S). We can see when initial deflection is increased, the dimensionless critical load of the pile is decreased correspondingly. But the effect of the initial deflection is small.

We also calculate the relationship between λ_{cr} and βl as shown in Fig.8, in which βl is a dimensionless parameter, and $\beta=\sqrt[4]{\frac{k}{4EI}}$. Because the critical load is the function of integer m^*, and m^* changes corresponding to the change of l as can be seen from eq. (17), the curve of λ_{cr} becomes a multiply bent curve for different m^*.

The relationships between P_{cr} and l for different N-value are calculated as shown in Fig. 9. Because of the nominal hinge at the point s_i and the different m^* due to the change of the length l, the solution P_{cr} becomes a wavy curve and has a limit value of $2\sqrt{EIk}\cos\theta$ for $l\to\infty$.

In Fig. 9, we can see the critical load increased when the N-value increased.

The effect of soil liquefaction on the P_{cr} is shown in Fig. 10. Here, the effect of the liquefaction is expressed as the reduction of the soil spring. It reveals that the effect of liquefaction is very significant, the critical load when the liquefaction occurs is very low compared with that when liquefaction does not occur.

5. Concluding remark

In this study we studied the single pile foundation with initial deflection. First, we derived the set of geometrically nonlinear equations for the problem. Then, we performed a series of parametric study to examine the critical load. The main results are summarized as follows:

(1) The critical load of pile foundation is correspondingly decreased when initial deflection is increased, but the effect of the initial deflection is small..

(2) We theoretically obtained the ultimate critical load for the infinite pile length. That is, $\lim_{l\to\infty} P_{cr}(l)=2\sqrt{EIk}\cos\theta_0$.

(3) The effect of liquefaction is very significant, the critical load when liquefaction occurs is very low compared with that when liquefaction does not occur.

REFERENCES

[1] Takaaki Miyasaka, Study on the Effectiveness of the High Ductility Aseismatic Joint Spliced Pile Subjected to Liquefaction-induced Large Ground Displacement, Doctor thesis of Yamaguchi-University, (1997.3).

[2] Hiromitsu Izumi, Dynamic Behavior of High Ductility Aseismatic Joint Spliced Pile Foundations during Strong Earthquake Motions, Doctor thesis of Yamaguchi University, (1999.3).

[3] L. Bjerrum, Norwegian Experiences with Steel Piles to Rock, Geotechnique, Volume 7, No.2, 73-96.(1957).

[4] C. A. Mascardi, Design Criteria and Performance of Micropiles, Symposium on Soil and Rock Improvement Techniques Including Geotextiles, Reinforced Earth and Modern Piling Methods, Bangkok, December, Paper D-3,(1982).

[5] J. Mandel, Flambement au Sein d'un Milieu Elastique, Annales des Ponts et Chaussees, (1936).

[6] W. G. K. Fleming, etc. Piling Engineering. 2st Edition. New York Surrey University Press, John Wiley and Sons, (1992).

[7] Japan Load Association, Specifications for Highway Bridges -Part V : Seismic Design, Maruzen, (1996)